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THE SEMICENTENNIAL CELEBRATION OF THE AMERICAN MATHEMATICAL SOCIETY—SEPTEMBER 6–9, 1938

The year 1938 is a very significant one in the life of the American Mathematical Society: it is the fiftieth anniversary of its birth.

It was in the summer of 1888 that a young man of twenty-three, Thomas Scott Fiske by name, returned from a period of study in England with the memory of inspiring afternoons at the London Mathematical Society fresh in his mind, and with an ambition to enjoy similar contacts in New York. By direct invitation he brought together a small group on November 24 of that year, and a month later this group organized itself into the New York Mathematical Society. Within three years the movement had spread widely enough so that it was feasible to begin the publication of a journal, the Bulletin of the New York Mathematical Society. Within three more years the national character of the enterprise became so apparent that the present more suitable names were chosen for both the Society and the Bulletin.

It is customary and seemly for societies, like individuals, to celebrate such anniversaries, to rejoice in the success of the years that are past, and to acquire new enthusiasm for the years ahead. Thus the summer meeting of the American Mathematical Society to be held in New York City in 1938 will be in the nature of a birthday party to celebrate this fiftieth anniversary. Like all successful birthday parties, it will have its moments of rejoicing and good cheer: an afternoon at Jones Beach, perhaps America's finest and certainly its least commercialized seaside resort; a trip up the Hudson to West Point (returning by moonlight) on one of the world's finest river boats, especially chartered for the occasion; the gayest banquet that the entertainment committee can provide; and many other diversions—more, indeed, than any one person can possibly attend.

Like all successful birthday parties, it will have its sentimental moments when honor is paid to those who conceived the Society and nurtured it through its tender years. Absence of such sentiments would be unthinkable, particularly at a gather-

ing distinguished by the presence of the founder himself and held on the campus of the University which has been so hospitable to the Society since its earliest days.

And like all successful birthday parties, it will have its serious moments: a program of nine invited addresses—by Professors J. W. Alexander, E. T. Bell, G. D. Birkhoff, G. C. Evans, E. J. McShane, J. F. Ritt, J. L. Synge, T. Y. Thomas, and Norbert Wiener; the usual opportunity for the presentation of shorter reports on current research; and ample opportunity for serious discussion on the personal plane.

From all three phases we shall hope to obtain a renewal of inspiration and strength for the coming years; for that splendid record of success which we shall celebrate is at once our greatest heritage and most sobering responsibility.

THORNTON C. FRY,
Executive Secretary of the Semicentennial Committee

HERBERT ELLSWORTH SLAUGHT—IN MEMORIAM

Among the members of our mathematical community in America one of the men most widely known by teachers and students of mathematics was Herbert Ellsworth Slaught. His lifelong devotion to his University and to the promotion of the study of mathematics, his skill as a teacher, his effective leadership in the mathematical organizations which he sponsored, and his influence with teachers of mathematics the country over, were remarkable. He was born July 21, 1861 and died on May 21, 1937 after a life of unusual success in the sequence of important undertakings to which he gave his enthusiastic support.

During his high school and college years Slaught's life was a difficult one. When he was thirteen years old his family lost the farm on Seneca Lake near Watkins, New York, which up to that time had been their home. It seemed a catastrophe, but Slaught himself has said that he would probably have spent his life working on the farm if it had not happened, and the farm apparently was not a very good one. At any rate in 1875 he and two older brothers of approximately college age found themselves in Hamilton, New York, with an older sister and an invalid mother to support, and with educations needing completion. The story of their success and of the cheerful cooperation of their family group would give encouragement to every young man who finds himself in similar circumstances. The two older boys became successful physicians, and Herbert graduated from Colgate Academy in 1879 and from Colgate University in 1883, each time as valedictorian of his class. He received his A.M. in 1886, and was honored with an Sc.D. in 1911, both from Colgate.

Soon after receiving his bachelor's degree Slaught was recommended by one of his professors for an instructorship in mathematics at Peddie Institute in Hightstown, New Jersey. As an instructor he was unusually successful from the start, and his administrative ability was soon recognized by his appointment to be assistant principal in 1886 and principal in 1889. But his real interest was in his mathematics and his teaching. He had married Miss Mary L. Davis, the instructor in music at Peddie, in 1885, and she sympathized with and encouraged his desire to enter the field of university mathematical work, even though such a course meant a serious sacrifice for them for some time to come. So in 1892 Slaught accepted a two-year appointment to one of the first three fellowships awarded by the Department of Mathematics at the University of Chicago, which was just then opening its doors. It was important both for the Department and for him that he had also a promise of summer quarter teaching to help out with his finances, since his performance in this teaching work early assured the Department of his skill as an instructor.

Thus began Slaught's long career at our University. He was relatively unprepared for graduate work in mathematics, but with his usual thoroughness he supplemented his college training intensively during the summer of 1892, and in the autumn was ready to carry on as a graduate student with success. In 1894, after the expiration of his term as a fellow, the fine quality of his teaching was evident and he was successively appointed reader, associate, assistant, and instructor, during the years 1894–97. In the winter quarter of 1898 he re-

ceived his Ph.D. degree, delayed because of his teaching work. He was made assistant professor in 1900, associate professor in 1908, professor of mathematics in 1913, and professor emeritus in 1931.

In the early years of the University of Chicago Slaught was responsible for entrance examinations in mathematics and was an official visitor of affiliated secondary schools. He also had charge of the mathematical meetings of the conferences of teachers of secondary schools which were then held at the University each summer quarter. Thus his influence and acquaintance with teachers of mathematics in the central west became even at that time very wide. During a sojourn in European universities in the college year 1902–3, and after a very conscientious debate with himself, he decided to devote his life to the promotion and improvement of the teaching of mathematics rather than to a research career. Upon his return to Chicago he affiliated himself enthusiastically with the newly formed Central Association of Science and Mathematics Teachers, and in 1907, at the suggestion of Professor L. E. Dickson, was made co-editor of the American Mathematical Monthly. From then on much of his time and interest outside of his departmental work was devoted to activities associated with the Monthly and the organization of the Mathematical Association of America and the National Council of Teachers of Mathematics. Slaught was also one of the first group of members of the Chicago Section of the American Mathematical Society, and he was a most efficient secretary of the Section from 1906 to 1916. His early recognition of the value of cooperation between the Society and the Association, and his effective encouragement of such cooperation, have constituted a most valuable service to mathematics in this country. He was at various times a member of the Council of the Society, trustee and president and honorary life president of the Association, honorary life president of the National Council, and honorary life member of the Central Association. A more detailed description of his important work in these organizations will appear in The American Mathematical Monthly in the near future.

Slaught had many contacts with the alumni of the University of Chicago and was much beloved by them. He was secretary of the Board of Recommendations of the University from 1903 to 1914, a period during which many of the policies now followed by its successor, the Board of Vocational Guidance and Placement, were formulated. He was secretary and unique guiding spirit of the Association of Doctors of Philosophy of the University for twenty-five years preceding his retirement in 1931. He was known everywhere among our alumni, so much so that wherever one went among graduates of our University one was sure to be called upon to answer the friendliest of inquiries concerning him.

But his interest in the welfare of others, and the community of those who had affectionate regard for him, extended beyond the University circle. In 1914 with a few of his associates in the Hyde Park Baptist Church he established the South Chicago Neighborhood House for the benefit of workers in neighboring steel mills and their families. The enterprise began with no asset except the enthusiasm of its sponsors, and now has the support of many interested individuals and organizations, a building of its own with trained workers, a membership of over 1000 persons, and an average monthly attendance of over 10,000 visitors and participants in its various activities. Slaught's devotion to this undertaking, through years of healthily increasing needs but

tardily growing resources, was typical of his determination and confidence in the things which he thought were good, and was the essential factor in its success. After the death of his wife he seemed to regard his work at the Neighborhood House as especially a memorial to her. The success and happiness of their partnership, which began with their marriage in 1885 and lasted until her death in 1919, could have no finer testimonial.

Slaught believed in the importance and the future of visual education. He was active in the organization of the periodical called the Educational Screen, and in 1922 made possible the publication of its first issue by his encouragement and financial support. Up to the time of his death his name appeared with the title "President" on the list of its Directorate and Staff.

Very few of Slaught's colleagues knew of his interest in the game called roque. On account of a physical infirmity dating from his boyhood he could not take part in more active forms of sport, but he and Mrs. Slaught played croquet often together in Hightstown before they were married, and when the game developed into roque they managed to have a court built on the grounds of Peddie Institute. In Chicago Slaught organized the Chicago Roque Club, with courts in Washington Park, and was its active president for twenty years and later honorary life president. Interest in the game grew in Chicago and elsewhere, and Slaught was a principal in the organization of the American Roque League which now controls the annual tournaments held in various sections of the country.

The preceding paragraphs have been concerned for the most part with Slaught's interests outside of his Department, but for us at Chicago his every day departmental activities were his most important ones. He was in many situations the ablest representative of our Department in our relations with the University and our students. His house was a common meeting place in which his friendly personality had a catalytic influence in promoting the happiest of relations between faculty and students. His classes, from freshman to beginning graduate grade, had a popularity which was notable and which withstood exceptionally the effects of the first two years of the depression up to the time of his retirement. Some of his colleagues who were enrolled in his courses in the early days testify as to their effectiveness, and I find in his correspondence letters from many former students which express appreciation of the clarity and friendliness of his teaching, letters which must have given him the greatest satisfaction. I should like to mention here the fact that one of the last things which he did for us, after his active work was over, was to increase greatly our collection of photographs of eminent mathematicians, now approaching one hundred in number, one of the best collections which I know. In the winter of 1935 he fell and broke his hip and was thereafter confined to his house, sometimes so ill as to be oblivious to affairs about him. But during many happier intervals his interest in the University and in mathematicians was as keen as ever, typical of his lifelong devotions. His was one of the careers which gave to our American mathematical circle a consciousness of its problems and a confidence in the importance of its work which have been essential to its development and success. We shall never again find quite such a colleague as Slaught, but his enthusiasm and devoted interest will long serve as ideals for us to emulate.

G. A. Bliss

University of Chicago

DE DONDER ON CALCULUS OF VARIATIONS

Théorie Invariantive du Calcul des Variations. By Th. de Donder. Paris, Gauthier-Villars, 1935. x+230 pp.

This book develops the purely formal parts of the calculus of variations. The title is justified by the study of the effect of transformations of coordinates and parameters upon the fundamental integral (1) and upon other expressions which occur in the theory. The author especially examines the conditions for the invariance of the integral under coordinate transformations.

The book is divided into three parts. The first of these consists of seven chapters, which contain the following material. Let V_τ be a piece of Euclidean $(x^1, \cdots, x^n)$-space depending on a parameter τ. Consider, furthermore, functions $y^i(x, \tau)$, $i=1, \cdots, m$, and a function F depending on τ, the x^ϵ, the y^i, and their partial derivatives up to a certain order c, and form the integral

$$(1) \qquad I(\tau) = \int_{V_\tau} F\left(x^i, y^i, \frac{\partial y^i}{\partial x^j}, \frac{\partial^2 y^i}{\partial x^h \partial x^j}, \cdots, \tau\right) dx^1 \cdots dx^n.$$

Then $I'(\tau)$ is expressed as the sum of an integral over V_τ and an integral over the boundary of V_τ. The method is extended to the case where V_τ is a piece of a surface of an arbitrary dimension in the n-space. The Stokes formula in a very general form and other classical results are derived as applications. Then the higher derivatives of $I(\tau)$ are represented in various forms. Five different expressions are given for $I''(\tau)$. These results are generalized by making the deformation of V depend on several parameters $\tau_1, \cdots, \tau_\sigma$ instead of one, yielding formulas for $\partial I(\tau_1, \cdots, \tau_\sigma)/\partial\tau_i$ and $\partial^2 I(\tau_1, \cdots, \tau_\sigma)/\partial\tau_i\partial\tau_j$. Chapter IV deals with integrals in the parametric form, that is, integrals $I(\tau)$ and $I(\tau_1, \cdots, \tau_\sigma)$ which are invariant under change of the variables $x^1, \cdots, x^n$ into any curvilinear coordinates. For $I(\tau_1, \cdots, \tau_\sigma)$ one finds, besides the usual conditions, additional ones that are rather complicated. The piece V_τ (dim $V_\tau < n$) is next represented implicitly as the intersection of manifolds $\Phi(x^1, \cdots, x^n) = 0$ or pieces thereof, and again conditions are given under which the integral is independent of the choice of the coordinates. The last chapter of the first part examines thoroughly the behavior of the expressions previously considered under transformations of the x^ϵ and shows that the methods developed here contain the main formulas of the absolute differential calculus as special cases.

The second part deals more properly with the calculus of variations. It starts with the definition of extremals for the parametric and the non-parametric cases, gives their invariant properties, and treats the theories of Lagrange and Hamilton-Jacobi. It is then occupied with the theorem of Jacobi concerning the relations between the Hamiltonian function and the integration of the canonical equations of a variation problem. First the author disposes of the classical case in which only one independent variable and only the first partial derivatives occur. Then generalizations in both directions are obtained, the calculations for the most general case becoming very complicated. In the

same way the classical theory of the Hilbert integral and the Weierstrass excess formula are treated. The end of the second part treats the equations of variation and generalizes H. Hahn's research on the connections between the theory of the second variation and the Weierstrass theory.

The third part contains applications to mathematical physics. It will be sufficient to name the subjects touched upon: the theory of gravitation, an electrically charged point mass in a gravitational and electrical field, the wave theory of light, and wave mechanics. Then the considerations again become purely mathematical for a time. That is, the theory of adjoint and self-adjoint systems of differential equations is briefly surveyed and applied to physical problems. The final chapter treats more closely the case where $F(x, y, \cdots, \tau)$ depends neither on τ nor on derivatives of higher order than the first, and applies these results to some problems of capillarity.

The reader of this review will miss some indications regarding the assumptions under which all these results hold. So did the reviewer when reading the book. He felt himself carried back to the heroic age of mathematics, when doubts had not yet retarded our steps. How smoothly runs the author's proof of the most general Stokes formula compared with the troubles which meager special cases cause to others! (The author actually compares his quite uncritical method with that in the book of Courant and Hilbert, p. 19.) The whole book can be regarded only as a collection of purely formal relations which hold locally under sufficiently strong conditions of regularity. I mention as an illustration that no actual use is ever made of conjugate points or of the concept of a field of extremals (the word is mentioned). Sometimes even the formal assumptions must be completed. For instance, §46, especially formula (543), could not be obtained if F really depended on τ, a possibility which the author did not exclude. One can see to what extent formalism prevails when the author says (p. 97), "on prend *par convention* $\delta'x=0$," and means that the end points of the extremal under consideration are moved *transversely*.

Nevertheless, since one aspect of the calculus of variations is treated so completely, the book will be of use for some questions of research. For instance, one finds many interesting identities.

There seem to be very few misprints and only unessential ones (for instance in formula (685)). This is astonishing in view of the numerous complicated formulas. The print is excellent. One would recommend a more extended use of the summation convention of the tensor calculus; and a list of the many symbols would be helpful.

HERBERT BUSEMANN

SHORTER NOTICES

Exercises d'Analyse. Vol. 4. By Gaston Julia. Paris, Gauthier-Villars, 1935. iv+230 pp.

This is a fourth and worthy companion volume to the other three volumes of *Exercises d'Analyse* by the same author. It is given over wholly to the solutions of fifty-five problems in partial differential equations of the first order. Again, as in the earlier volumes, it is a set of exercises which figure in the program of certification of the differential and integral calculus at such institutions as the École Normale Supérieure and the Sorbonne, and which constitute further material supplementary to Goursat's classic *Cours d'Analyse.* Students preparing for the examinations of the licence and the agrégation will, of course, work through these volumes.

Some few of the problems are of such character that they might be treated in an elementary first problem course in differential equations,—exercises, for example in solving: (1) systems of the form

$$\frac{dx}{dt} = f_1(x, y, z), \quad \frac{dy}{dt} = f_2(x, y, z), \quad \frac{dz}{dt} = f_3(x, y, z);$$

(2) total equations $Pdx+Qdy+Rdz=0$; (3) linear partial differential equations $Pp+Qq=R$; (4) the more general equation $f(x, y, z, p, q)=0$, where the methods of Lagrange and Charpit may be used. For the most part, however, the problems are of a theoretical nature and suitable only to give to more mature students. Many of the exercises are solved in more than one way, and thus the established spirit of the series is maintained. Most of them are taken from past examinations, Paris predominating, some even dating back to the 1870's. More new material might have been included.

Problems with boundary conditions are treated only slightly and singular solutions not at all. Numerous applications are made to the field of geometry but none to physical problems. Free use is made of methods employing geometry, with and without coordinates, and also of the complex variable.

The printing is well done, on paper of improved quality.

C. O. OAKLEY

Cours de Cinématique. By Gaston Julia. Edited by Jean Dieudonné. Second Edition. Paris, Gauthier-Villars, 1936. 161 pp.

The writings of French scientists in general are undoubtedly outstanding for their grace and elegance of style. There is a smoothness and ease of development, with nevertheless a completeness and rigor, which makes them fascinating for reading and study. This little volume continues that tradition.

The author begins with the kinematics of a point and develops the elementary theory by a combination of vectorial and analytic methods.

In the second chapter the motion of a rigid body is considered, the broad

general principles being given. The helicoidal motion of a set of moving axes is discussed and then it is shown that the most general motion of a rigid body can be described as a helicoidal motion. The helicoidal movement gives the velocities but not the accelerations. This is pointed out, but it might have been useful to account for this difference by pointing out the distinction between vectors and vector quantities.

Chapter III deals with the relative motions of different systems of axes and the composition of relative motions, and after developing the general principles makes various applications, including treatments of the methods of Poinsot and Roberval. The discussions here are principally for the two dimensional case, but in Chapter IV the application to sets of moving coordinate axes in space is made.

The remaining chapters deal with more detailed study of the motions of rigid bodies. Chapter V is devoted to plane motion. The theory of the instantaneous centre is developed, and also the method of describing continuous plane motion by the rolling of one curve on another. The formulas of Euler and of Euler-Savory are obtained, and the graphical construction of Savory for relating instantaneous centres is given. Several applications are made, including a discussion of various types of epicycloids.

In Chapter VI the motion of a rigid body with one point fixed is considered. This is done by studying the motion of a sphere, with centre at the fixed point and radius arbitrary, relative to a fixed sphere having the same centre and radius. This permits of the description of the motion by means of curves on these spheres rolling on each other, just as in the case of plane motion.

The final chapter deals with the general motion of a rigid body. Use is made of the line complexes formed by the normals to the trajectory of a point and by the tangents to a trajectory, and motion is described by the rolling of one surface on another.

All told, the author has given a very thorough and well organized development of his subject. The notation used is simple and consistent. Vector and coordinate modes of expression are intermingled, and the free and natural choices that have been made render the discussion easy to follow.

There are slight inconsistencies in two or three of the Figures but these are not serious, and the number of typographical errors noted was small.

J. W. CAMPBELL

Statistical Mechanics. By R. H. Fowler. Cambridge, University Press; New York, The Macmillan Company, 1936. Second Edition. 864 pp.

The publication of a new enlarged edition of Fowler's treatise on statistical mechanics brings forcibly to the reader's attention the rapidity with which work in this field has progressed since 1929. There is a wealth of new material presented both from the theoretical and experimental side and the order of presentation has been changed by introducing the quantum mechanical point of view from the outset and omitting all reference to the older Bohr theory.

The mathematical methods utilized in the first edition remain essentially unchanged, and as these have been reviewed so ably in this journal by M. H. Stone (vol. 39 (1933), p. 850) it is hardly necessary to comment further on

these points. It will be disappointing to many, as it is to the reviewer, that Fowler has not included a critical discussion of the fundamental postulates of statistical mechanics, especially since the whole subject is approached from a quantum mechanical viewpoint. To be sure, an exposition of the work of von Neumann in this connection would be "somewhat lengthy and would form a portion necessarily out of tune with the rest of this monograph," but the more elementary treatments instigated by Jordan and Pauli would seem to be well worthy of inclusion.

The book itself is a veritable mine of information, the theoretical treatment and the discussion of experimental data being well mixed throughout the work. The author has also ventured somewhat outside the field of statistical mechanics to present the theory of those non-equilibrium flow problems in which many of the results of equilibrium theory can be applied as a good approximation.

In view of the tremendous scope of the book it seems appropriate to outline the contents. After an introductory chapter stating the fundamental assumptions of the theory, the next three chapters are concerned with the equilibria of perfect gases, crystals, and any general system obeying classical laws. Chapter V discusses problems of dissociation and evaporation and Chapter VI treats the connections between statistical mechanics and thermodynamics in detail, merging into the material of the next chapter which presents the Nernst heat theorem and chemical constants. The three following chapters extend the theory of imperfect gases, equations of state, and a survey of intermolecular forces as derived from imperfect gas equations and allied crystal data. Chapter XI covers the field of thermionics and the simpler aspects of conduction of electricity and allied effects in metals, and this is followed by a chapter on magnetic and dielectric properties of matter in bulk, including ferromagnetism. Chapter XIII attempts to develop the theory of liquids. The next three chapters contain the theory as applied to the high temperatures inside and outside the stars, whereas the next three contain detailed studies of the laws to which the mechanisms of interactions must conform in order to preserve the equilibrium laws. The next chapter is devoted to fluctuation phenomena, and the final chapter summarizes some recent work, principally along the lines of cooperative phenomena.

The book has been produced in the superlative style of the University Press and stands as an invaluable contribution to the literature on statistical mechanics.

N. H. FRANK

Généralités sur les Probabilités. Variables Aléatoires. By M. Fréchet. Paris Gauthier Villars, 1937. xvi+308 pp.

This book belongs to the series bearing the general title *Traité du calcul des probabilités et de ses applications*, edited by E. Borel. It is the first book of Fascicule III of Tome I of this series. Tome I is entitled: *Les principes de la théorie des probabilités*, Fascicule III: *Recherches théoriques modernes sur la théorie des probabilités*. This Fascicule III is divided into two books. The first book, which we have here under consideration, is mainly devoted to the

"aleatory variables," the second book will deal with the method of arbitrary functions and of the chain theories in probability. We are promised, besides, a treatment of several related investigations in Fascicule III of Tome IV, entitled *Compléments divers*. We thus have the possibility of an extensive and expert discussion of these theoretical aspects of the theory of probability, which have been the object of much recent research. This gives the book a unique and important position.

Fréchet's monograph is divided into two parts. The first covers "généralités sur les probabilités" and is introductory. It contains mainly certain rigorously established theorems on total and compound probabilities for the case of dependent and independent events, finite or infinite in number. The second part, on *Les variables aléatoires*, forms the bulk of the book and is, broadly speaking, a rigorous theory of moments. The starting point is the monotone function $C(x)$, $0 \leq C(x) \leq 1$, representing the probability that a numerical event X be less than x. The different moments can be expressed as Stieltjes integrals with the aid of this function $C(x)$; for example, the mean as

$$\overline{X} = \int_{-\infty}^{+\infty} x \, dC(x).$$

Various theorems on moments of independent and dependent aleatory variables receive a rigorous discussion. A chapter is devoted to the inequality of Bienaymé and its generalizations, which gives the author the opportunity to point to the importance of Bienaymé's work of 1853. More than a hundred pages deal with the different ways in which convergence of series of aleatory variables can be understood. A part of this has already appeared in vol. 8 of *Metron*, 1930, by the same author. The reader who has not been able to follow recent work in the different periodicals will find, especially in this part of the book, a veritable treasure of investigations. A supplement deals with monotonic functions, and a note with a new property of Laplace's second law. The book ends with a note by Paul Lévy on the distance of two aleatory variables and the distance of two probability laws.

The type of work discussed and moulded into a unity in this book is characterized by the names of Borel, Cantelli, Copeland, Cramer, Fréchet, Khintchine, Kolmogoroff, P. Lévy, Slutsky, and others. Their approach dates back, essentially, to Borel's paper of 1909 in the Rendiconti del Circolo Matematico di Palermo on the "probabilités dénombrables." This paper placed the mathematical theory of probability definitely in the general framework of Lebesgue's theory of measure, and made it possible to use the results of modern integration theory for the general study of moments and of infinite series of aleatory variables. Fréchet's book is the first of its kind to give us a good survey of the field. But also the more practical statistician, who does not want abstract information so much as a good treatment of the deeper lying theorems of ordinary statistics, such as Tschebychef's theorem, the law of large numbers, the laws of moments for dependent variables, the correlation coefficient, will find ample information in this book, the more so as the only modern books of a similar character in this particular respect, those of S. Bernstein and R. von Mises, are in Russian and German respectively.

D. J. STRUIK

Pseudosphärische, Hyperbolisch-Sphärische und Elliptisch-Sphärische Geometrie. By F. Schilling. Leipzig and Berlin, Teubner, 1937. viii+240 pp.

The writer of this review has previously reported on several of Professor Schilling's works for the Bulletin, see for instance vol. 38 (1932), p. 335. The present volume is a further elaboration of the *Pseudo-sphere and non-euclidean geometry*, with the purpose of making this fascinating subject even more attractive to the ordinary trained teacher and student than in the preceding volumes.

In this task Professor Schilling, of the Technische Hochschule of Danzig, has succeeded admirably and I can repeat in praise only what I have said in my previous reviews.

ARNOLD EMCH

L'Emploi des Observations Statistiques. Méthodes d'Estimation. By G. Darmois. (Actualités Scientifiques et Industrielles, No. 356; Statistique Mathématique, exposés publiés sous la direction de Georges Darmois, I.) Paris, Hermann, 1936. 29 pp.

The first part of this booket deals with problems of estimating unknown parameters of frequency distributions from n observations by finding a point in the space of the parameters by the use of a point in the space of the observations, thus giving the best information concerning the position of the "true point."

Part two presents definitions of the distance of an estimation from the true value, and limits of estimations. Comparisons are made of precisions of laws for the mean and median for certain probability laws, together with a comparison of the standard errors of the mean deviation and the standard deviations. The importance of the ratios of two standard deviations is shown.

The next section gives a geometric meaning of the maximum of the function of points in the parametric and observational spaces and shows how to determine the principal part of the standard deviation of an estimation of a parameter which furnishes the maximum of this function. The maximum of this function is compared with R. A. Fisher's maximum of likelihood. The geometric interpretation is very interesting.

The last part defines exhaustive estimation which is equivalent to R. A. Fisher's sufficient estimation. Comparisons are made of two estimations of an unknown parameter by means of the sizes of standard deviations. The quantity $g=E[\partial \log f/\partial m]^2$, plays an important role in the last two sections; f is a frequency function.

W. D. BATEN

Méthode des Caractéristiques pour l'Intégration des Équations aux Dérivées Partielles Linéaires Hyperboliques. By Mlle. Hélène Freda. (Mémorial des Sciences Mathématiques, Fascicule LXXXIV.) Paris, Gauthier-Villars, 1937. 7+82 pp.

The following list of chapter titles gives a good notion of what to expect from this pamphlet: Préface de V. Volterra; Chapitre I. Équations hyperboliques: problèmes aux limites et variétés caractéristiques; Chapitre II. Inté-

gration des équations hyperboliques à deux variables (méthode de Riemann); Chapitre III. Intégration de l'équation

$$\theta(u) = \frac{\partial^2 u}{\partial x_1^2} + \frac{\partial^2 u}{\partial x_2^2} - \frac{\partial^2 u}{\partial x_3^2} = f(x_1, x_2, x_3)$$

(méthode de Volterra); Chapitre IV. Quelques extensions de la méthode de Volterra; Chapitre V. Intégration, par la méthode des caractéristiques, de l'équation générale, linéaire, hyperbolique, du type normal; Bibliographie.

The exposition is restricted in general to the solution of second order partial differential equations of the hyperbolic type

$$\sum_{ij} A_{ij} \frac{\partial^2 u}{\partial x_i \partial x_j} + \sum B_i \frac{\partial u}{\partial x_i} + C - U = f,$$

in which case the *characteristics* used for generating the integral are real. In the general case these characteristic manifolds are *conoids*. When the A_{ij} are constants the characteristic conoid reduces to a cone, and in other more special cases to straight lines.

The pamphlet is the result of a course of lectures given by Mlle. Freda at the University of Rome, 1931. The treatment is classical following the researches of d'Adhémar, Tédoue and Coulon, Picard, and Volterra, with extensions and modifications by Mlle. Freda. Consideration is also given the applications to mechanics and physics.

The exposition is clear but very compact. One unfamiliar with the subject would profitably read a more elementary treatment. As Volterra points out in the preface and as the bibliography indicates, the preparation of this monograph was long and difficult, depending on the researches of many writers spread over a number of years.

Mlle. Freda is to be congratulated not only on her perseverance but also on the excellence of her accomplishment.

V. C. Poor

Elemente der Funktionentheorie. By K. Knopp. Berlin, de Gruyter, 1937. 144 pp.

This little book is Number 1109 in the well-known and highly regarded Sammlung Göschen. The author restricts himself to the treatment of only the simplest parts of the theory that are important for a more detailed and extensive study. The selection of material has been made with the discrimination of a scholar and is written with the clarity of style we have come to expect from the pen of the author.

The thirteen chapters are grouped in five parts: 1. Complex numbers and their geometric representation. 2. Linear functions and the circular transformation. 3. Aggregates and sequences. Power series. 4. Analytic functions and conformal representation. 5. The elementary functions.

To a reader already acquainted with the subject it may seem strange that the integral calculus of complex functions is omitted entirely. However, this enables the author to treat the topics included with amazing thoroughness in the small compass of 135 pages of text. One wishing to go further will find ex-

cellent material in the author's two volumes, *Funktionentheorie*, Erster Teil; Zweiter Teil. These are Numbers 668 and 703 in the Sammlung Göschen.

A student entering upon the study of a new subject needs many exercises to test his understanding. There are none in this book, but the author's two volumes, *Aufgabensammlung zur Funktionentheorie* (Nos. 877, 878) will provide ample material.

G. E. RAYNOR

Sur la Théorie Mathématique des Jeux de Hasard et de Réflection. By René de Possel. (Actualités Scientifiques et Industrielles, No. 436; Conférences du Centre Universitaire Méditerranéen de Nice, publiés sous la direction de M. Paul Vallery, I.) Paris, Hermann, 1936. 41 pp.

Chapter I describes how the three factors, reflection, hasard, and wile, enter the "game of society," by use of several games.

The next chapter explains games of batons. Piles of batons or sticks are before the players who can remove, when their turn comes, a number of sticks less than or equal to a given number; the player who takes the last stick loses. More complicated cases of this game are analyzed by means of expressing the numbers involved in the system of notation with 2 as base.

The third chapter states a theorem concerning games of combinations in which one player wins or there is a draw.

Chapter IV contains the usual definitions of probability, mathematical expectation, and equitable games. Mathematical expectation is applied in detail to the roulette wheel.

In the last chapter the author defines the "game of society" and outlines how to study the influence of reflection, the influence of hasard, and the influence of wile, which enter into games. Maxima and minima of the mathematical expectations of the player have an important role in this part. Applications of these maxima and minima are made for several games.

W. D. BATEN

Biologie Mathématique. By V. A. Kostitzin. Paris, Libraire Armand Colin, 1937. 223 pp.

In an appreciative preface, Vito Volterra points out that while numerous books have been published in recent years on the applications of mathematics to biology, it remained for M. Kostitzin to produce a synthetic and didactic work drawing together the researches on this subject. As is stated by the author, this book is fundamentally different from the manuals of mathematics with similar titles that have been prepared for students of biology. While he recognizes the utility of such books for reference purposes, the author is not enthusiastic about selecting certain chapters of one science for the benefit of workers in another. He says that each science has its peculiar language and logic and that it is only by preserving these that the applications of science can have their full force. Carrying this thought further, the author says that if a biologist needs chemistry he had better study that science and not read a few chapters especially adapted for biologists. Similarly, to be able to apply the methods of mathematics it is necessary to study what constitutes the science of mathematics—its ideas—and not some of the processes of calculation.

The author states that mathematics has entered the natural sciences through the door of statistics, but that statistics yields to analysis here as in all of the rational sciences. The role of statistics is to blaze the trail, to establish certain empirical laws, and to facilitate the passage from statistical to analytical variables.

This book devotes a part of one chapter to a discussion of frequency polygons and correlation. Then the author gives an example of two related series of values with a correlation coefficient of zero, and concludes that while the failure of the method of correlation in this case does not prove that it is useless, it indicates limitations in its usefulness and gives an added argument for the analytical method.

The book is principally a study of the growth and decline of various biological "populations." The method is to construct one or a set of differential equations or integro-differential equations representing variations of the population with time. As a rule these equations cannot be solved in their general forms, but the author finds that many conclusions of importance can be drawn from the solution of the equations in particular cases, or for critical values of parameters involved, and from the simpler equations that are recognized as only approximately correct.

Among the problems discussed are the growth of "closed" populations, the effect of poisoning due to metabolic products, relations between different species competing for the same food supply, species that feed on each other, symbiosis, and parasitism. Other chapters discuss the growth of organisms, embryonic and post-embryonic growth, forms of living beings and evolution.

The book shows, as Volterra points out in the preface, that the author is acquainted with the most modern resources of analysis, that he has an aptitude and ability at solving mathematical problems that have resisted earlier students, and a grasp of fundamental problems in biology.

R. B. ROBBINS

NOTES

A colloquium on the theory of probability will be held at Geneva, Switzerland, October 11–16, 1937, under the auspices of the University of Geneva. This is one of a series of colloquia (Conférences Internationales) which have been held, for several years, at this university. The list of speakers includes the following: S. Bernstein, P. Cantelli, H. Cramér, E. L. Dodd, W. Feller, B. de Finetti, M. Fréchet, V. Glivenko, W. Heisenberg, E. Hopf, B. Hostinsky, C. Jordan, A. Kolmogoroff, P. Lévy, R. von Mises, J. Neymann, N. Obrechkoff, O. Onicescu, G. Pólya, E. Slutsky, J. F. Steffensen, H. Steinhaus, A. Wintner. The titles of the papers by Professors Dodd and Wintner are, respectively, *Certain coefficients of regression or trend associated with largest likelihood*, and *Distribution functions of sums of independent random variables.*

The Royal Academy of Sciences of Turin has elected as corresponding members Professor E. Bompiani, of the University of Rome, and Professor G. Krall, of the University of Naples.

Dr. Sydney Goldstein, lecturer in mathematics at the University of Cambridge, was elected a Fellow of the Royal Society of London at the meeting held on May 6, 1937.

Professor Paul Montel, of the Sorbonne, has been elected a member of the Section of Geometry of the Paris Academy of Sciences, in succession to the late Professor Edouard Goursat.

Among the delegates appointed by the United States Department of State to the Descartes Congress held in Paris, France, early in August, were Dean G. D. Birkhoff, chairman of the delegation, and Professors Arnold Dresden and L. C. Karpinski. The occasion was the three hundredth anniversary of the publication of Descartes' *Discourse on Method.*

At the dinner of the National Academy of Sciences held on April 27, 1937, the James Craig Watson Medal was awarded to Professor Emeritus E. W. Brown, of Yale University, for his work on lunar theory.

At the recent centennial celebration of Mount Holyoke College, the degree of doctor of science was conferred on Professor Anna P. Wheeler, of Bryn Mawr College.

The following have been appointed National Research Fellows in Mathematics for the year 1937–38: R. P. Boas, Jr., D. M. Dribin, Dorothy Manning, A. E. Taylor, C. B. Tompkins, H. S. Zuckerman. This list includes renewals.

Dr. F. W. Dresch has been appointed Florence Noble Travelling Fellow of the University of California.

Dr. L. A. Pipes has been appointed research fellow at the University of Wisconsin.

Dr. O. F. G. Schilling has been appointed Johnston Scholar at the Johns Hopkins University for 1937–38.

Associate Professor Amato, of the University of Messina, has been promoted to a professorship.

Associate Professor Cherubino, of the University of Pisa, has been promoted to a professorship.

Associate Professor Gugino, of the University of Palermo, has been promoted to a professorship.

Dr. Rudolf Iglisch has been appointed to an assistant professorship in mathematics at the Technical School of Aachen.

Dr. Alfred Klose has been appointed professor of applied mathematics and director of the institute for applied mathematics at the University of Berlin, in succession to Professor K. T. Vahlen.

Professor Hellmuth Kneser, of the University of Greifswald, has been appointed to a professorship at the University of Tübingen.

Dr. Gottfried Köthe has been appointed to an assistant professorship in mathematics at the University of Münster.

Dr. Wilhelm Maier has been appointed to an assistant professorship in mathematics at the University of Freiburg.

Associate Professor Maroni, of the University of Modena, has been promoted to a professorship.

Associate Professor Masotti, of the University of Milan, has been promoted to a professorship.

Associate Professor Mattioli, of the University of Catania, has been promoted to a professorship.

Dr. K. T. Vahlen, professor of applied mathematics at the University of Berlin, has been made an honorary professor at the University of Greifswald.

Dr. G. A. Baker has been appointed instructor in mathematics and assistant statistician at the Experiment Station, College of Agriculture, University of California.

Associate Professor J. P. Ballantine, of the University of Washington, has been promoted to a professorship.

The title of A. B. Dinwiddie Graduate Professor of Mathematics has been conferred upon Professor H. E. Buchanan, of Tulane University.

Dr. Scott Buchanan has been appointed dean of Saint John's College, Annapolis.

Dr. E. A. Cameron, of the University of North Carolina, has been promoted to an assistant professorship.

Acting Professor E. J. Camp, of Yankton College, has been appointed to an associate professorship at Macalester College, St. Paul.

Dr. Elizabeth M. Cooper has been appointed chairman of the mathematics department of Hunter College High School.

Dr. H. P. Doole, of the University of Nebraska, has been promoted to an assistant professorship.

Mr. E. D. Eaves, of the University of Tennessee, has been promoted to an assistant professorship.

Assistant Professor D. A. Flanders, of New York University, has been promoted to an associate professorship.

Dr. A. H. Fox, of Union College, has been promoted to an assistant professorship.

Dr. Israel Halperin has been appointed to a Benjamin Peirce Instructorship at Harvard University.

Mr. Ernest Hawkins, of the United States Naval Academy, Annapolis, has been promoted to an assistant professorship.

Professor Alfred Hume, of the University of Mississippi, who was reported, in the July issue of this Bulletin, page 463, to have retired with the title of professor emeritus, has informed the editors that this announcement is incorrect. He retired from the Chancellorship of the University in 1935, and is now Chancellor Emeritus and head of the department of mathematics, on full time.

Assistant Professor A. R. Jerbert, of the University of Washington, has been promoted to an associate professorship.

Associate Professor R. A. Johnson, of Brooklyn College, has been promoted to a professorship.

Professor Edward Kasner, of Columbia University, has been appointed to the Adrain Professorship of Mathematics.

Dr. S. C. Kleene, of the University of Wisconsin, has been promoted to an assistant professorship.

Assistant Professor C. I. Lubin, of the University of Cincinnati, has been promoted to an associate professorship.

Dr. S. W. McCuskey, of the Case School of Applied Science, has been promoted to an assistant professorship in mathematics and astronomy.

Professor H. B. MacDougal, of South Dakota State College, has been appointed acting head of the department of mathematics.

Mr. Roy MacKay has been appointed to an associate professorship at Teachers College, Silver City, N. M.

Dr. G. B. Price has been appointed to an assistant professorship at the University of Kansas.

Dr. E. J. Purcell, of the University of Arizona, has been promoted to an assistant professorship.

Dr. I. J. Schoenberg, of Colby College, has been promoted to an assistant professorship.

Professor W. T. Stratton, of Kansas State College, has been appointed head of the department of mathematics.

The following appointments to instructorships are announced: University of Arkansas: Dr. E. G. H. Comfort; Brooklyn College: Mr. Jack Wolfe; Case School of Applied Science: Dr. R. F. Rinehart; Harvard University: Mr. R. F. Clippinger, Mr. M. P. Fobes, Mr. A. D. Hestenes, Mr. D. T. McClay, Mr. H. E. Robbins, Mr. A. Spitzbart; University of Maryland: Dr. O. K. Sagen; University of Missouri: Dr. R. J. Michel; University of North Carolina: Dr. Nathan Jacobson; Oregon State Agricultural College: Dr. Henry Scheffé; Randolph Macon Woman's College: Dr. Virginia Modesitt, Miss E. R. Schneckenberger; University of Southern California: Dr. F. A. Butter, Jr.; Texas Technological College: Mrs. O. L. Miller, Mr. H. E. Woodward; Utah State Agricultural College: Dr. M. T. Bird; Wayne University: Dr. E. W. Paxson.

Dr. G. A. Maggi, formerly professor in the University of Milan and a member of the Royal Academy of the Lincei, died on June 12, 1937 at the age of eighty-one years. He was noted especially for his work on general conceptions in old and new mechanics.

Professor Emeritus Vittorio Martinetti, of the University of Messina, died on November 2, 1936 at the age of seventy-seven years.

Professor Adalrigo Masoni, of the Engineering School of Naples, died on September 29, 1936 at the age of seventy-six years.

Professor R. D. Beetle, of Dartmouth College, died on July 9, 1937 at the age of fifty-one years. He had been a member of the Society for twenty-seven years.

Mr. Henry Moir, chairman of the finance committee and for many years president of the United States Life Insurance Company, died on June 8, 1937 at the age of sixty-six years.

ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross-references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

314. Professor R. P. Agnew: *On existence of linear functionals defined over linear spaces.*

A functional $r(x)$ defined over a linear space E is called an r-function (over E) if there exists a linear functional $f(x)$ with $f(x) \leqq r(x)$ for all $x \in E$. In order that $r(x)$ may be an r-function, it is necessary and sufficient that, for each $x \in E$, the greatest lower bound of $\sum_{k=1}^{n} r(t_k x_k)/t_k$, for all finite sets of positive numbers $t_1, \cdots, t_n$ and elements $x_1, \cdots, x_n \in E$ with $x_1 + \cdots + x_n = 0$, be non-negative. Moreover, if $r(x)$ is an r-function, then the greatest lower bound of $\sum r(t_k x_k)/t_k$, for all finite sets of positive numbers $t_1, \cdots, t_n$ and elements $x_1, \cdots, x_n \in E$ with $x_1 + \cdots + x_n = x$, is a p-function (Banach, *Théorie des Opérations Linéaires*, p. 28) $p^{(r)}(x)$ with $p^{(r)}(x) \leqq p(x)$; and $p^{(r)}(x)$ is the largest p-function with $p(x) \leqq r(x)$. (Received August 2, 1937.)

315. Mr. H. W. Alexander: *The role of the mean curvature in the immersion theory of surfaces.*

This paper falls naturally into two parts. In the first part an expression is obtained for the second fundamental tensor of a surface in terms of the mean curvature and its first and second derivatives, and the first fundamental tensor $g_{\alpha\beta}$ and its derivatives. These expressions are then substituted in the equations of Codazzi to obtain two third order differential equations in the mean curvature, which are completely integrable, and which are necessary and sufficient conditions that a quantity K_m be the mean curvature of a surface whose first fundamental tensor $g_{\alpha\beta}$ is given. In the second part, singularities of immersion of intrinsically regular surfaces are discussed, and are shown to be points where the mean curvature is infinite. Under a simple basic supposition, it is proved that the locus of such points is a regular curve E, "the edge of regression," and that the coordinates of the surface in the neighborhood of E are regular functions of the arc length along E and the square root of the arc length along geodesics orthogonal to E. Also, in the neighborhood of E there exists a second surface, applicable to the first and meeting it cuspidally. (Received July 31, 1937.)

316. Professor Reinhold Baer: *Groups with abelian central quotient group.*

The method employed throughout this investigation of groups which have abelian central quotient groups may be described as follows. If G is such a group, then preference is given to a subgroup S, situated between the central and the commutator group of G. Now G and S determine $G^*=G/S$ and certain invariant relations between the abelian groups G^* and S. The subgroups S and G^*, and these relations between G^* and S, are invariants of G if S is in particular either the central or the commutator group of G. In general these invariants do not completely determine the structure of G. They form a complete set of invariants, however, if G^* is a direct product of cyclic groups, and this is the only hypothesis restricting the generality of this method. (Received July 27, 1937.)

317. Professor Reinhold Baer: *Groups with preassigned central and central quotient group.*

It is well known that not every abelian group is the central quotient group of a suitable group, and those abelian groups with a finite number of generators which are central quotient groups have been characterized before. This note is devoted to a characterization of those pairs of groups C, Q which are central and central quotient group, respectively, of a suitable group. This problem is completely solved under the hypothesis that Q is a direct product of cyclic groups. The necessary and sufficient conditions for the existence of a group whose central is the abelian group C and whose central quotient group is the direct product Q of cyclic groups essentially relate the number of cyclic direct factors of Q whose order is n and the number of independent elements of order n in C. (Received July 27, 1937.)

318. Professor Harry Bateman: *Halley's methods for the solution of equations.*

About 1694 Edmund Halley gave two general methods for solving equations by iteration, one based on a rational expression for the second approximation in terms of the first and a second one in which the corresponding expression is irrational. It was this work that led to the discovery of Taylor's expansion of a function in a power series. In the present paper the convergence of the first process is considered, use being made of known results for the corresponding Newtonian process. Some extensions of the method are considered. (Received July 23, 1937.)

319. Dr. E. F. Beckenbach: *Remarks on the problem of Plateau.*

From the result that the problem of Plateau is solvable for an arbitrary finite-area-spanning Jordan contour, alternate proofs are given of the known result that the problem is solvable for an arbitrary Jordan contour. (Received July 17, 1937.)

320. Mr. Garrett Birkhoff: *Additive functions on Boolean algebras.*

Let B be any Boolean algebra, and let Σ denote the set of bounded additive real functions $f(x)$ defined on B. By the "total variation" of any $f\in\Sigma$, is meant the l.u.b. $|f|$ of the finite sums $\sum_{k=1}^{n}|f(x_k)|$, over disjoint elements $x_k\in B$. Relative to $|f|$ as norm, Σ is a Banach space, and the non-negative functions $p(x)$ with $p(I)=1$ form a closed convex bounded set Δ in Σ. Further, if T is a linear operator on Σ satisfying $T(\Delta)\leqq\Delta$, then $|T(f)|\leqq|f|$; that is, T is an isometry or a contraction. Moreover, if some iterate T^n of T satisfies $pT^n\geqq\lambda p_0$, $(\lambda>0)$, for all $p\in\Delta$, then T^n is a *uniform* contraction of Δ, and shrinks distances in ratios $\leqq(1-\lambda)$. It follows, generalizing a fixed-point theorem of Picard, that (1) there is exactly one $r_0\in\Delta$ left fixed by T, (2) powers T^k of T contract Δ to r_0 with the rapidity of a geometrical progression. If countable sums and products are defined in B, the completely additive functions $f(x)$ are a closed linear subspace Γ of Σ, and so the above results hold in Γ. Further, an argument of Hahn shows that, given $p_0\in\Delta$, the $f\in\Gamma$ for which $p_0(x)=0$ implies $f(x)=0$ are associated with *density* functions, whose integrals with respect to the measure p_0 are the original f. (Received July 10, 1937.)

321. Mr. Garrett Birkhoff: *On dependent probabilities.*

A matrix of transition probabilities is a linear operator T on the convex space Δ of "distribution functions." Hence if one introduces "total variation" as norm, transition probabilities can be interpreted geometrically. For example, if T is deterministic, it is isometric. Independent probabilities induce a projection onto a point. Moreover, in the "stochastic" case, $pT^n\geqq\lambda p_0$ for all p and some n, $\lambda>0$, p_0; hence iterations of T contract Δ towards a unique "stable" distribution—generalizing the usual theory of *discontinuous* stochastic processes. Further, each *continuous* stochastic process (that is, each continuously observed system whose transitions occur by occasional jumps) is associated with a continuously defined *rate of transition*, and conversely. In particular, each transition probability of change in a unit interval of time is associated (via the exponential-logarithmic series) with one and only one constant rate of transition. To establish this, one requires an infinitesimal calculus for linear operators on Banach space. (Received July 10, 1937.)

322. Professor E. T. Browne: *Sets of conjugate matrices.*

If M is a given square matrix of order n, and if $M_1, \cdots, M_{t-1}$ are $t-1$ matrices of the same order which possess the properties (1) the M's are commutative in pairs, and (2) $(\lambda-M)(\lambda-M_1)\cdots(\lambda-M_{t-1})\equiv f(\lambda)$, wherein λ is a scalar and $f(\lambda)$ a scalar polynomial in λ; then $M_1, \cdots, M_{t-1}$ are said to constitute a *set of conjugates* to M. If t is the smallest integer for which an identity of the above type holds, the set of conjugates is called a *reduced* set. Sets of conjugates have been studied by Taber, Franklin, Sokolnikoff, Pierce, Hermann, and others. In this paper a thorough study is made of sets of conjugates, and more general sets are obtained than those given hitherto. In particular, a special study is made of *cyclic* sets. (Received July 10, 1937.)

323. Dr. J. W. Calkin: *Self-adjoint differential operators of elliptic type. Resolvent kernels.*

The author defines in the complex Hilbert space $\mathfrak{L}_2(E)$, E an open set in the (x,y)-plane, a linear transformation T: $Tf=L(f)\equiv-(pf_x)_x-(pf_y)_y+qf$; p_{xx}, p_{yy}, and q bounded measurable, $p>0$, and $\Im(q)=0$ on every bounded closed set interior to E. The domain of T is characterized by explicit local conditions on its elements and the implicit condition $\int_E|L(f)|^2dE<\infty$. The following results are obtained: $T=\bar{T}$, $T\geqq T^*$. The deficiency-index of the symmetric transformation T^* is (n, n) and all of the cases $n=0, 1, 2, \cdots, \aleph_0$ occur. If H is a maximal symmetric extension of T^*, H^* its adjoint, and λ is a point of the resolvent set of H, there exist kernels $G(P, Q; \lambda)$, $G^*(P, Q; \bar{\lambda})$ of Carleman type such that, for every g in $\mathfrak{L}_2(E)$, $(H-\lambda I)^{-1}g(P)=\int_E G(P, Q; \lambda)g(Q)dQ$ and $(H^*-\bar{\lambda}I)^{-1}g(P)=\int_E G^*(P, Q; \bar{\lambda})g(Q)dQ$. The identity $\overline{G(Q, P; \lambda)}\equiv G^*(P, Q; \bar{\lambda})$ holds at every point of $E\times E$, $P\neq Q$. At the points $P=Q$, $G(P, Q; \lambda)$ becomes logarithmically infinite. The equations $L[G(P, Q; \lambda)]=\lambda G(P, Q; \lambda)$ and $L[G^*(P, Q; \bar{\lambda})]=\bar{\lambda}G^*(P, Q; \bar{\lambda})$, Q (P) fixed and $L(G)$, $L(G^*)$ calculated with respect to the coordinates of P (Q), are satisfied everywhere on $E-Q$ $(E-P)$. (Received August 3, 1937.)

324. Professor H. V. Craig: *On tensors relative to the extended point transformation.*

The principal object of this paper is to explain, by means of the concept extensor (tensor relative to the extended point transformation) and a process called contraction over a reduced range, the structure of certain of the invariants and vectors of higher order geometry. (Received July 27, 1937.)

325. Professor H. B. Curry: *Some properties of formal deducibility.*

In a deductive system $\mathfrak{S}$ let those intuitive propositions with whose deduction one is concerned in the formal developments be called elementary propositions; in the Hilbert theory, for example, these are of the form "$\mathfrak{A}$ is provable," where $\mathfrak{A}$ is a specific formula. Taking these propositions as basic, this paper is concerned with the system $\mathfrak{T}$ of propositions obtained from them by the usual logical sentence-combinations. These combinations are defined intuitively; for example, "$A\supset B$" means "B is provable in the system obtained by adjoining A to $\mathfrak{T}$." The problem of ascertaining what logical calculus is valid when its variables are interpreted as unspecified propositions of $\mathfrak{T}$ is considered. If the positive operations are the usual ones, and "not-A" is "A implies every elementary proposition," then the Heyting calculus is valid. With negation of another sort related to non-deducibility, the minimal calculus of Johansson with the addition (for certain $\mathfrak{S}$'s) of the law of excluded middle is valid. The notion of necessity is defined by considering a family of systems $\mathfrak{S}$; the resulting calculus bears a relation to the Heyting calculus similar to that which the calculus of C. I. Lewis bears to the classical one. (Received August 2, 1937.)

326. Dr. D. M. Dribin (National Research Fellow): *Normal extensions of quartic fields with the symmetric group.*

Let B be a normal algebraic number field of degree 6 whose group is the symmetric group on 3 letters. The author finds in the present paper necessary and sufficient conditions that $N=B(\mu_1^{1/2}, \mu_2^{1/3})$, ($\mu_1$ and μ_2 in B), be normal of degree 24 whose group is the symmetric group on 4 letters; these conditions are described by the effect of the automorphisms of B upon μ_1 and μ_2 and are derived by simple class-field theoretic considerations. The author also proves the interesting result that every total-imaginary field B of the type described above can be extended to a field N of the type described with μ_1 and μ_2 taken as units in B. These results are applied to questions of existence of Hilbert subgroup series in normal extensions of quartic fields with the symmetric group, lately discussed by the author. (Received July 29, 1937.)

327. Professor O. J. Farrell: *On the representation of bounded analytic functions by sequences of polynomials.*

Let $f(z)$ denote a function analytic and bounded in a region G of the plane of the complex variable z. Let $d(f, P)$ denote the diameter of the cluster set of $f(z)$ at a point P on the boundary B of G. Let $D(f, B)$ denote the maximum of $d(f, P)$ for all points P on B. The problem considered is concerned with the existence of polynomials $p_n(z)$, $n=1, 2, \cdots$, which converge to $f(z)$ in G in such a way that as $n\to\infty$ the superior limit of the least upper bound of $[|f(z)-p_n(z)|, z \text{ in } G]$, does not exceed $D(f, B)$. A complete solution of the problem is not reached in this paper. It is found, however, that a sufficient condition for the representation of every such function by such a sequence of polynomials is that G be a limited Jordan region. Necessary conditions for the representation of every such function by such a sequence are that G be limited and, if simply connected, that B divide the extended plane into just two regions with B as their common boundary; if G is multiply connected, it is necessary that the closed region $G+B$ be the complement of an infinite region. (Received July 29, 1937.)

328. Dr. Aaron Fialkow: *Conformal transformations and the subspaces of a Riemann space.*

A conformal transformation, $d\bar{s}=\lambda ds$, which maps one Riemann space upon another induces a conformal correspondence between the respective subspaces of each. This induced mapping is studied in detail. A typical result is that two minimal varieties correspond if and only if the gradient $\partial\lambda/\partial x^i$ lies in the varieties. Other results do not depend upon the given conformal map but hold for a conformal Riemann space. Thus the lines of curvature always correspond. The hypersurfaces, $\lambda=$constant, play a dominant role in this work. Groups of conformal transformations (in particular, iterated maps) are also considered. (Received July 26, 1937.)

329. Dr. Kurt Friedrichs: *On the definition of differential operators.*

In order to apply the theory of self-adjoint linear operators to differential

operators, it is not sufficient to consider functions which are differentiable in the ordinary sense. Instead of using concepts of Lebesgue's theory, the author gives a quite different method of extension, equally applicable to ordinary and partial differential operators and directly concerning those properties which are essential for self-adjointness. (Received August 3, 1937.)

330. Dr. H. H. Goldstine: *A multiplier rule in abstract spaces.*

The author formulates a problem of minimizing a functional, defined on a region of several normed linear spaces, in a class of points satisfying certain operational equations. This problem is designed so as to include the problem of Bolza in the calculus of variations, the problem of minimizing a function of several variables subject to side conditions, and numerous other special cases. Analogues of the Euler-Lagrange equations and of the transversality conditions are obtained by a method which is essentially a generalization of the ordinary proof given in the calculus of variations. (Received July 26, 1937.)

331. Dr. J. A. Greenwood: *Variance of a general matching problem.*

In this problem, t distinct groups of s identical cards each are matched against an equal-sized deck composed of $i_1, i_2, \cdots, i_t$ cards of each symbol. From a table giving the relative frequencies of hits and misses of all pairs of cards in the latter deck, the average correlation between all pairs of cards is found. Substitution in a well-known formula then gives the variance. It is also proved that the variance is a maximum when $i_1=i_2=\cdots=i_t=s$. Then $\sigma^2=n^2(t-1)/t^2(n-1)=n\sigma_b^2/(n-1)$ where $n=ts$ and σ_b^2 is the variance of the corresponding binomial case. (Received August 3, 1937.)

332. Mr. Philip Hall and Mr. J. K. Senior: *The groups of order* 64. Preliminary report.

The authors have made independently and by different methods a complete determination of the groups of order 64. Besides the 11 abelian groups, they have found 117 groups of class 2, 114 groups of class 3, 22 groups of class 4, and 3 groups of class 5. The total of 267 groups differs from that given in previous publications. A detailed account of the classification and properties of these groups is in course of preparation. (Received August 6, 1937.)

333. Dr. W. L. Hutchings: *On a canonical form of the system of differential equations defining a ruled surface.*

In this paper it is shown that if a ruled surface R_{yz}, defined by the system $y''+p_{11}y'+p_{12}z'+q_{11}y+q_{12}z=0$, $z''+p_{21}y'+p_{22}z'+q_{21}y+q_{22}z=0$, has distinct non-linear flecnode curves, the system may always be reduced to a canonical form for which $q_{12}=q_{21}=p_{11}+p_{22}=0$, $p_{12}=p_{21}=1$. Wilczynski's fundamental theorem that if the invariants $\theta_4, \theta_{4\cdot 1}, \theta_9, \theta_{10}$ are given arbitrarily ($\theta\theta_{10}\neq 0$) they determine a ruled surface to within a projective transformation, is established without quadratures. The intersection of corresponding osculating planes of the flecnode curves of R_{yz} determine a ruled surface $R_{\eta\zeta}$ covariant with R_{yz}. The canonical form here obtained is applied to a study of the correspondences

existing between these two surfaces. In this study, intersector curves, first defined by E. P. Lane, are of primary importance. At two points of each ruling of R_{yz} the asymptotic and intersector directions coincide. The invariant condition that the locus of these points be asymptotics, also that the involute curves of R_{yz} be asymptotics, is given. Conditions that the flecnode curves of the two surfaces correspond, that the flecnode curves of $R_{\eta\xi}$ be plane curves, and that $R_{\eta\xi}$ be a quadric surface are obtained. Some theorems of A. F. Carpenter concerning the flecnode and complex curves of a ruled surface are generalized and extended. (Received August 2, 1937.)

334. Dr. D. H. Hyers: *On functional equations in linear topological spaces.*

An existence theorem for differential equations in linear topological spaces has been proved by E. W. Paxson (see abstract 43-3-232), making use of the Brouwer-Tychonoff fixed point theorem. These theorems, however, are non-effective and tell us nothing about the uniqueness of the solution. In the present paper a new type of Lipschitz condition makes possible the use of successive approximations in proving existence and uniqueness theorems for differential and other functional equations in linear topological spaces. Point set theoretic methods are used extensively. An application is made to the theory of infinite systems of ordinary differential equations in real variables. (Received August 3, 1937.)

335. Professor M. H. Ingraham: *On certain equations in matrices whose elements belong to a division algebra.*

Roth, Franklin, the author, and others have studied the matrix equation $P(X)=A$, where P is a polynomial with coefficients in a given field. The natural generalization to matrices and polynomials with elements in a division algebra D leads to the study of the equation $P(X)\odot\xi=Q(A)\odot\xi(\xi)$, where the $\odot$-operation is defined as in an article by the author and Dr. M. C. Wolf (Transactions of this Society, vol. 42 (1937), pp. 16–31). The solution of this equation reduces to that of a system of equations with coefficients in the centrum. The solution of any one of these is found subject to factoring polynomials with coefficients in the centrum of D into irreducible factors with coefficients in D. The problem of finding the simultaneous solutions of such a system is still under consideration. (Received August 2, 1937.)

336. Dr. Nathan Jacobson: *A note on topological fields.*

Let F be a locally compact, separable, and totally disconnected (quasi-) field and J an anti-automorphism of F, that is, $(a+b)^J=a^J+b^J$, $(ab)^J=b^Ja^J$. Then F has degree one or two over its centrum C. If F has characteristic $\neq 2$ and J is involutorial ($a^{J^2}=a$ for all a in F) and $c^J\neq c$ for some c in C, then $F=C$ is commutative. Similar results hold for F_n a complete matrix ring with coefficients in F. An important lemma used in the proof is the following generalization of a result due to F. K. Schmidt: any automorphism or anti-automorphism of F is continuous (in fact isometric in a "naturally" defined metric). (Received July 19, 1937.)

337. Professor Fritz John: *On the solution of the differential equation* $u_{x_1x_1}+u_{x_2x_2}=u_{x_3x_3}+u_{x_4x_4}$.

In a recent paper in the Mathematische Annalen, L. Asgeirson proved for a solution $u(x_1, x_2, x_3, x_4)$ of this "ultrahyperbolic" equation the mean value theorem: $\int_0^{2\pi}u(x_1+r\cos\phi, x_2+r\sin\phi, x_3, x_4)d\phi=\int_0^{2\pi}u(x_1, x_2, x_3+r\cos\phi, x_4+r\sin\phi)d\phi$ for any x_1, x_2, x_3, x_4, r. In the present paper u is interpreted as a function of the lines in xyz space with equations $x=x_1+x_3-(x_2-x_4)z$, $y=x_2+x_4+(x_1-x_3)z$. Asgeirson's theorem then states that for every hyperboloid of one sheet the values of the integrals of u over the two families of generating lines are equal. With the help of this theorem, it is proved that every solution of the differential equation satisfying certain regularity conditions may be represented as the line integrals of a function $f(x, y, z)$. (Received July 29, 1937.)

338. Dr. Gertrude S. Ketchum and Dr. P. W. Ketchum: *On a certain class of non-linear expansions of an arbitrary analytic function.*

It is shown that any function $f(x)$ analytic at the origin has the expansion $f(x)=f(0)+\sum b_nF_n(a_nx^n)$, where the functions $F_n(\omega)$ and the constants b_n are given in advance, subject to the following four conditions: (1) $F_n(\omega)$ has a simple zero at the origin, (2) $|F_n(\omega)|\leqq M_n$ for $|\omega|\leqq\rho$, where $M_n=O(n^a)$ for some a, (3) $|b_n|^{1/n}$ is bounded for all n, (4) there exist constants C, a, and σ such that $0\leqq|b_m|\leqq Cn^a\sigma^{n/m}|b_n|$ for all integers n and all integers $m<n$ which divide n. Certain infinite product expansions follow from this theorem. The particular case $F_n(\omega)=\omega(1-\omega)^{-1}$ has been studied by Feld (Annals of Mathematics, vol. 33 (1932), pp. 139–142). (Received July 29, 1937.)

339. Professors M. S. Knebelman and Walther Mayer: *Parallel vector fields.*

An n-dimensional affinely connected space is defined by n N-rowed square matrices $\Gamma_{j\alpha}^{i}(C_{x_0}^{x})$, $\alpha=1, \cdots, n$; $i, j=1, \cdots, N$, where $C_{x_0}^{x}$ is an arc joining the point x_0 to x. Here C is assumed to be piecewise differentiable and the functions Γ bounded, continuous, and independent of parametrization. A vector y_0^i is displaced, parallel to itself, along C if $dy^i/dt=\Gamma_{j\alpha}^{i}y^jdx^\alpha/dt$; then $y^i=y_0^jF_j^i(C_{x_0}^{x})$ where the matrix F is given by an infinite series whose terms are multiple line integrals. The series is convergent and F is non-singular in some neighborhood of x_0. The fundamental property of F is $F(C_{x_0}^{x_1})F(C_{x_1}^{x})=F(C_{x_1}^{x})$. For the existence of N independent parallel vector fields the equations $\partial y^i/\partial x^\alpha=\Gamma_{j\alpha}^{i}y^j$ must be completely integrable. To obtain these conditions the problem is reduced to two dimensions and all curves are reduced to those made up of coordinate lines. By a limiting process the authors then express these conditions by the vanishing of a set of matrices whose elements are point functions. In the case of a connection whose matrices are point functions of class C', these reduce to the vanishing of the curvature tensor. (Received July 20, 1937.)

340. Professor R. E. Langer: *The expansion theory of ordinary differential systems of the first order.*

This paper deals with the differential system $y'(x)-[\rho q(x)+r(x)]y(x)=0$, $\alpha y(a)=\beta y(b)$, in which the coefficients, except for the parameter ρ, are real, and in which $q(x)$ may change its sign. The theory for the expansion of arbitrary functions in solutions of such a system, as heretofore given (see M. H. Stone, Transactions of this Society, vol. 26 (1924), page 335), has been based on the orthogonality of the solutions over the interval (a, b). Its results are sharply in contrast with the analogous ones for differential systems of higher order. A new theory is here given. It is based on the orthogonality of the solutions over any point set Δ on (a, b), upon which the function $Q(x)\equiv\int_a^x q(x)dx$ fulfills, just once, the congruence $Q(x)\equiv t(\text{mod } Q(b))$, for almost every t on the range $0<t<Q(b)$. The results are much more nearly in agreement with those for other differential systems, and the necessary hypotheses on $q(x)$ are much less restrictive. These hypotheses are, in brief, that $q(x)$ be summable, that there exist an open point set Δ on which $Q(x)$ fills the requirement above, and on each component interval of which $q(x)$ is almost everywhere of one sign. (Received July 17, 1937.)

341. Dr. D. C. Lewis: *Invariant manifolds near an invariant point of unstable type.*

In the case of surface transformations the existence of an invariant curve in the neighborhood of an invariant point of unstable type and containing the invariant point was proved by Poincaré, Hadamard, and Birkhoff. Here the author gives a generalization of Hadamard's proof to the case of transformations in multiple dimensional spaces. (Received July 30, 1937.)

342. Dr. D. C. Lewis: *Representation of states in quantum mechanics by entire functions.* Preliminary report.

The impulse and coordinate operators p and q are replaced by the conjugate operators $x=p+iq$ and $y=p-iq$. Confining the problem for simplicity to the case of one degree of freedom, one is led to the representation of states by those entire functions $f(z)$ such that $\left|f(z)\exp\left(-z^2/(4h)\right)\right|$ is bounded for real z and $\left|f(z)\exp\left(z^2/(4h)\right)\right|$ for pure imaginary z. Here $2\pi h$ is Planck's constant. A certain subclass of this class of entire functions forms a Hilbert space, in which the inner product of two elements is defined in terms of the coefficients in the power series of the corresponding two entire functions. Any complete set of eigenstates (including those of p and q themselves) which do not lie in a Hilbert space are "relatively" normalized with respect to each other by the requirement that the corresponding representatives of p and q be Hermitian. A general theorem to the effect that an arbitrary entire function of the type indicated can be represented in the form $\int_{-\infty}^{+\infty}\phi(z, \lambda)d\alpha(\lambda)$, where $\phi(z, \lambda)$ represents an eigenstate of an assigned Hermitian operator with eigen-value λ, has not been obtained. Upon such an expansion a probability theoretic interpretation may be built up in almost the usual way. (Received July 30, 1937.)

343. Mr. L. L. Lowenstein: *Linear equations with an infinity of unknowns.*

This paper establishes criteria that a set of equations $\sum_{k=1}^{\infty} a_{pk}x_k = c_p$, $p=1, 2, \cdots, m \leqq \infty$, either have solutions (1) such that for each $\epsilon>0$, $\sum |x_k|^2 < \epsilon$ or (2) such that for all solutions $\sum |x_k|^2 \geqq \theta > 0$. For $m < \infty$ let $\{a_{1k}\}, \cdots, \{a_{mk}\}$ be a set of linearly independent sequences of numbers. If there exist numbers $\mu_1, \cdots, \mu_m$, not all zero, such that $\sum_{k=1}^{\infty} |\sum_{p=1}^{m} \mu_p a_{pk}|^2 < \infty$, then the set of sequences will be said to be *quadrilinearly dependent*; otherwise *quadrilinearly independent*. For $m < \infty$ quadrilinear independence and dependence of the set of sequences of coefficients of the equations under consideration are criteria for solutions of type (1) and type (2), respectively. If $m = \infty$ then it is said that the set of sequences is quadrilinearly dependent if some finite subset is quadrilinearly dependent. Quadrilinear independence is a necessary condition for solutions of type (1) if $m = \infty$. It is not sufficient; however, it is sufficient to ensure the existence of a certain solution in a generalized sense of type (1). Necessary and sufficient conditions for the quadrilinear dependence of sets of sequences are established. (Received August 3, 1937.)

344. Professor R. G. Lubben: *Perfectly compact Hausdorff spaces in which a normal space may be embedded.*

Let S be a normal Hausdorff space and let M be the aggregate of all T's, where T is a perfectly compact Hausdorff space of which S is a sub-space and in which S is dense. The author defines the term "boundary point of S" and establishes the existence of a sub-aggregate N of M such that (1) if T_N is an element of N, the points of $T_N - S$ are boundary points of S and (2) each element of M is topologically equivalent to some element of N. There exists an element T_ω of N such that (1) the points of $T_\omega - S$ are "atomic" boundary points of S, and (2) if T is an element of M, there exists in T_ω an upper semi-continuous collection, K_T, of mutually exclusive closed point sets, the points of S being elements of K_T, such that T is topologically equivalent to the decomposition space of T_ω which is determined by K_T. (Received July 10, 1937.)

345. Mr. J. K. L. MacDonald: *Bounds for parameters in n-noded solutions of Sturm-Liouville equations.*

The form $dp(x)/dx \cdot dy(x)/dx + q(x)y(x) = 0$, ($p(x)>0$, $q(x)>0$ in (x_1, x_2)), is considered with respect to the relations between parameters in $q(x)$, and the number of nodes for $y(x)$ in (x_1, x_2). A first order differential equation for a phase angle is developed by use of a transformation involving two adjustable functions. By suitable choices of the adjustable functions in various cases, the first order equation is expressed in forms convenient for approximate integration (with bounds for errors). Means for including end conditions are also developed. (Received July 30, 1937.)

346. Dr. A. J. Maria: *Displacement of equilibrium point of Green's function for an annulus.*

In this note there is given an estimate of the displacement of the equilibrium

point of Green's function for an annulus as the pole moves along a fixed diameter from the inner to the outer boundary. There are also results on the magnitude of the displacement for small and large annuli. (Received August 6, 1937.)

347. Professor W. H. McEwen: *A note on an extension of Bernstein's theorem.*

In a recent paper (American Journal of Mathematics, vol. 59 (1937), pp. 295–305) the author obtained an extension of Bernstein's theorem to arbitrary sums of Birkhoff type, of the form $S_N(x)=\sum_{i=1}^{N}a_iu_i(x)$ in which the u's are the first N characteristic solutions of a general regular nth order linear differential system $L(u)+\lambda u=0$, $W_i(u)=0$, $(i=1, 2, \cdots, n)$, defined on $a\leqq x\leqq b$. Assuming that $|S_N(x)|\leqq L$ on (a, b), the author found that $|S_N'(x)|\leqq QNL$ uniformly on any interior interval $(a+\delta, b-\delta)$, Q being a constant independent of N, and an example was cited to show that this is the best result that can be obtained in general. In particular cases, however (as for example in the Fourier case), the limit QNL can be extended to the whole interval (a, b). In this paper general additional hypotheses are considered under which this extension to the whole interval can be made. (Received July 26, 1937.)

348. Professor A. D. Michal: *General conformal differential geometry.*

General Riemannian differential geometry with Banach coordinates has already been studied by the author on several occasions. The theory centering around a conformal curvature form is developed in the present paper with the aid of additional postulates. (Received August 3, 1937.)

349. Professor A. D. Michal: *General projective differential geometry.*

This paper deals with a new chapter in the author's general differential geometries with Banach coordinates. The normed ring of linear transformations in the Banach space is subjected to postulates insuring the existence of a suitable "contraction." The existence of a projective curvature form is demonstrated and it is then proved (barring a few singular cases) that a necessary and sufficient condition that a general geometry with a symmetric linear connection be locally projectively flat is that the projective curvature form vanish. The existence theorems for completely integrable abstract differential equations play a fundamental role in these studies. (Received August 3, 1937.)

350. Dr. A. P. Morse and Dr. J. F. Randolph: *Gillespie linear measure for point sets.*

Several definitions of the linear measure for a point set A have been given. Of these linear measures, the one $L(A)$ by C. Carathéodory seems to be the most adequate generalization of curve length. In this paper, however, a plane set A is constructed such that its projection X on the x-axis is $0\leqq x\leqq 1$ and Y is $0\leqq y\leqq 1$, but nevertheless $L(A)$ is only unity instead of at least $2^{1/2}$. A different linear measure $G(A)$ is defined and named Gillespie linear measure after

the late Professor D. C. Gillespie who suggested similar definitions to the authors. With $|X|$ the outer Lebesgue measure of X, it is proved for any plane set A and any axes that $G(A) \geqq (|X|^2+|Y|^2)^{1/2}$. If A consists of all points of a simple rectifiable arc, the length of the arc and $G(A)$ are proved equal. For such sets $G(A)=L(A)$. Always $L(A) \leqq G(A)$ and it is easily shown that $G(A) \leqq \pi L(A)$. Furthermore it is proved that $G(A) \leqq (\pi/2)L(A)$. This relation is shown to be the best possible by constructing a set for which the equality sign holds. Also, Gillespie area measure is defined for sets in three or more dimensions and a basic relation between "G-area" and "G-length" is established, the analogue of which has not been proved for Carathéodory's measures. (Received August 3, 1937.)

351. Dr. D. L. Netzorg: *An extension of Markoff's comparison theorems for zeros of orthogonal polynomials.*

Let $\rho_1(t)<\rho_2(t)<\cdots<\rho_n(t)$ be the zeros of the nth polynomial $P_n(x, t)$, orthogonal with respect to the non-negative weight function $p(x, t)$ over a fixed interval $a \leqq x \leqq b$. If $(1/p)\partial p/\partial t$ is a convex function of x over (a, b), it is impossible to have $i<j$ while $\rho_i(t)$ is an increasing, and $\rho_j(t)$ a decreasing function of t. If, furthermore, $a=-h$, $b=+h$, and p is an even function of x over (a, b), $\rho_i^2(t)$ is an increasing function of t $(i=1, 2, \cdots, n$, except for $i=m$, if $n=2m-1$, where $\rho_i(t)=0)$, so that the positive zeros increase, and the negative zeros decrease, as t increases. Taking $p(x, t)=(h^2-x^2)^t p(x)$, where $p(x)$ is an even function over $(-h, +h)$, one obtains a theorem due to Stieltjes (Acta Mathematica, vol. 9 (1886), pp. 385–400). Markoff's proof of the same theorem (Mathematische Annalen, vol. 27 (1886), pp. 177–182) is erroneous, as Professor Szegö has pointed out. (Received August 4, 1937.)

352. Dr. D. L. Netzorg: *Certain inequalities for functions of exponential type belonging to L along the real axis.*

If the entire function $f(z)=O(e^{|z|})$ is such that $f(x)$ belongs to $L_2(-\infty, +\infty)$, then, for every $\alpha \geqq 0$, $\int_{-\infty}^{+\infty}|t^\alpha f(t)|\,dt \geqq C_\alpha\left|\int_{-\infty}^{+\infty} f(t)dt\right|$, where $C_\alpha>0$ depends only on α. (Received August 4, 1937.)

353. Dr. D. L. Netzorg: *The mean number of solutions of $\phi(x)=n$.*

Denote by $g(n)$ the number of solutions of $\phi(x)=n$, where ϕ is the Euler ϕ-function. Then $\lim_{n\to\infty}(1/n)\sum_{\nu=1}^{n} g(\nu)=\prod_p(1+1/p(p+1))=1.9432+$, the product extending over all primes. Similar asymptotic evaluations are obtained for $\sum \nu^\alpha \phi^\beta(\nu)$ summed over $\nu^\alpha \phi^\beta(\nu) \leqq n$. Generalizations are given to any numerical function $\psi(n)$ such that $\psi(mn)=\psi(m)\psi(n)$ when $(m, n)=1$, and $\psi(p^k)=p^{k-1}\psi(p)$ for all primes p. A recurrent method is given for constructing a table of solutions of $\phi(x)=n$. (Received August 4, 1937.)

354. Professor E. G. Olds: *The distribution of sums of squares of rank differences for the case of eight individuals.*

In a previous paper (abstract 43-1-48), the author presented the exact dis-

tributions for sums of squares of rank differences for the cases of numbers of individuals from two to seven, and compared them with approximations obtained by use of the Gaussian and Pearson type II functions. The present paper gives the exact distribution for the case of eight individuals and indicates the degree of accuracy when the exact distribution is replaced by one of the above-mentioned functions in testing the significance of rank correlation. (Received July 30, 1937.)

355. Dr. A. E. Pitcher and Dr. W. E. Sewell: *Existence theorems for solutions of differential equations of non-integral order.*

Liouville and Riemann considered a derivative in which the order α may assume any real value. In this paper the authors establish an abstract existence theorem of the sort proved by Birkhoff and Kellogg. It is applied to prove existence theorems for solutions of differential equations of the form $y^{\alpha} = \phi(x, y)$. In the above equation α is positive, ϕ is a known function of two real variables, y is an unknown function of x, and y^{α} denotes the derivative of order α. (Received August 3, 1937.)

356. Professor Tibor Radó: *On continuous transformations of bounded variation in the plane.*

Let T: $x = x(u, v)$, $y = y(u, v)$ be a continuous transformation, defined in the square Q_0: $0 \leqq u \leqq 1$, $0 \leqq v \leqq 1$. If T is of bounded variation, then it possesses a certain area-derivative $D(u, v)$ which exists almost everywhere in Q_0 and is summable there (see Banach, Fundamenta Mathematicae, vol. 7 (1925), pp. 225–236). Schauder (Fundamenta Mathematicae, vol. 12 (1928), pp. 47–74) introduced a topologically defined function $i(u, v)$ and derived a number of results which show that the product $J(u, v) = i(u, v)D(u, v)$ may be considered as a generalization of the Jacobian of a pair of functions. In order to secure the summability of $J(u, v)$, Schauder (loc. cit.) assumed that $i(u, v)$ is bounded. In a previous paper (Fundamenta Mathematicae, vol. 27 (1936), pp. 201–211), the present author proved that $J(u, v)$ is always summable if the transformation T is absolutely continuous in the sense of Banach. In the present paper, it is shown that this result remains true for transformations of bounded variation in the sense of Banach. (See Banach, loc. cit., for the terminology.) (Received July 23, 1937.)

357. Professor J. F. Ritt: *On certain points in the theory of algebraic differential equations.*

The following topics are treated: forms in several unknowns; pairs of forms; essential manifolds composed of one solution; an approximation theorem; equations in two unknowns, of the first order. (Received July 28, 1937.)

358. Mr. L. B. Robinson: *A quasi-analytic function which satisfies a functional equation.*

A generalized Lambert series has been studied (M. C. Garvin, American Journal of Mathematics, vol. 58 (1936), pp. 507–513), giving an example of a

quasi-analytic function. Consider the series $f \equiv F_0 + \lambda F_1 + \cdots + \lambda^n F_n + \cdots$ wherein $F_n \equiv \int_0^{\xi} (1-r^2)^{-1} \int_0^r (1-s^4)^{-1} \cdots \cdot \int_0^u (1-z^{2n})^{-1} dz^{2n-1} dy^{2n-2} \cdots ds^3 dr$. The function f has a natural boundary on the unit circle. Since f also converges elsewhere for sufficiently small values of λ, f is quasi-analytic. It also satisfies the functional equation $u'(\xi) = \lambda u(\xi^2)/(1-\xi^2)$. (Received April, 1937.)

359. Professor A. E. Staniland: *Analytic transformations in euclidean space of 2n dimensions.*

A euclidean $2n$-space, S_{2n}, is employed as a representation of a complex n-flat, Σ_n, coordinates of mated points of the two spaces subject to the correspondence $z_k = x_{2k-1} + ix_{2k}$, $(k = 1, 2, \cdots, n)$. Sets of analytic functions defined in Σ_n are represented by analytic hypersurfaces S_{2n}. An analytic transformation of S_{2n} is one that leaves invariant the class of analytic surfaces. A transformation $x_r = f_r(x_1', x_2', \cdots, x_{2n}')$, $|\partial f_r/\partial x_s'| \neq 0$ and $\partial f_r/\partial x_s'$ continuous, is analytic if and only if it is equivalent to one of the two analytic types, $A: z_k = \phi_k(z_1', z_2', \cdots, z_n')$, $B: \bar{z}_k = \phi_k(z_1', z_2', \cdots, z_n')$. The jacobians, j and J, of the real and complex transformations are related by the identities $j \equiv \pm J\bar{J}$, the negative sign holding for $B(n \text{ odd})$. Types A and B are *normal* at P_0 in Σ_n when the roots of $[\partial\phi_k/\partial z_j']$ are all distinct at P_0. Then the *differential* transformations A_1 and B_1, of matrix $[\partial\phi_k/\partial z_j']$, regarded formally as transformations of the neighborhood of P_0 into itself, leave invariant n complex line elements E_1^μ through P_0. With the E_1^μ as new complex axes, A_1 and B_1 assume the multiplicative forms $dz_\mu = r_\mu e^{i\theta_\mu} dz_\mu'$ and $d\bar{z}_\mu = r_\mu e^{i\theta_\mu} dz_\mu'$. The latter reduces to $d\bar{z}_\mu = r_\mu dz_\mu'$ when the new axes are used. The E_1^μ correspond to n real invariant plane elements Π_μ through Q_0, the representative of P_0; hence the mapping of S_{2n} upon itself by A and B is conformal with respect to the *components* in Π_μ of elementary arcs originating at Q_0. (Received August 7, 1937.)

360. Dr. A. H. Taub: *Spin representation of inversions.*

A subgroup of the conformal group of a flat n-space is the direct product of the rotation group and the group of two elements composed of the inversion and the identity. The spin representation of the former is well known. The spin representation of the latter is obtained. The representation is then used to study the behavior of Dirac's equation under conformal transformations. The results of Dirac and Veblen are obtained from this viewpoint. It is further shown that if the mass term in Dirac's equation is set equal to zero, the equation is conformally invariant. Since this is done in the neutrino theory of light, it is thus proved that this theory has the same invariance property as the Maxwell theory. (Received July 26, 1937.)

361. Dr. A. H. Taub: *Spin representations of groups of motion of Riemannian spaces of constant curvature.*

Riemannian spaces of constant curvature can be imbedded in flat spaces of one higher dimension. The group of motion of the former is then represented by a subgroup of the euclidean group of the flat space. The spin representation of the latter is known, and hence one obtains a spin representation of the group

of motion of any Riemannian space of constant curvature. In obtaining the representation explicitly, it is found that two coordinate systems are equally useful. The relation between these is also discussed. (Received July 26, 1937.)

362. Dr. R. M. Thrall and Dr. Josephine H. Chanler: *Ternary trilinear forms in the field of complex numbers.*

Equivalence and general equivalence are defined for the general trilinear form $\sum_{h=1}^{k}\sum_{i=1}^{l}\sum_{j=1}^{m} a_{hij}x_h y_i z_j$, where k, l, m are the smallest numbers of x's, y's, and z's in terms of which the form can be expressed, and a_{hij} are complex numbers. In *Metabelian groups and trilinear forms* (presented to the Society, April 9, 1937), Thrall has shown that the problem of determining equivalent classes of forms is abstractly identical with that of classifying the matrices $M_x(x)=(\sum a_{hij}x_h)$ under certain multiplications and transformations; in that article the classification was carried out for the ternary case for forms in a $GF(p)$. In the present paper the main problem is the determination of generally-equivalent classes of ternary forms in the complex field. Here a geometric method is more illuminating. The conditions imposed by the form upon the curves $|M_x(x)|=0$, $|M_y(y)|=0$, $|M_z(z)|=0$, are studied; in particular, the projective invariants of the curves are shown to be invariants of the form, and certain Cremona transformations relating them are investigated. A table of generally-equivalent classes is given, and a typical member of each class is exhibited. (Received July 30, 1937.)

363. Mrs. C. C. Torrance: *Superposition on monotonic functions.*

In this paper the following theorem is proved. Let $g(x)$ be a general monotonic function. Then if $f(y)$ has the Baire property in the restricted sense, $f[g(x)]$ has the Baire property in the restricted sense. Whether or not $f(y)$ has the Baire property, $f[g(x)]$ can (1) have the Baire property in the restricted sense, or (2) have the Baire property, or (3) fail to have the Baire property. (Received July 16, 1937.)

364. Professor W. J. Trjitzinsky: *Theory of non-linear singular differential systems.*

In this work the author investigates (in the complex plane, in the vicinity of the singular point $t=\infty$) properties of solutions of systems (A) $t^{-p}y_j^{(1)}(t)=a_j(t, y_1, \cdots, y_n)$, $j=1, \cdots, n$, integer $p\geqq 0$, wherein second members are analytic in $t, y_1, \cdots, y_n$ at $(t=\infty, y_1=0, \cdots, y_n=0)$. The particular case of (A) when the second members are independent of t and $p=0$ is of importance in dynamics. This problem, called (B), has been also investigated at length in the complex t-plane. Finally, systems (C) $\lambda^{-p}y_j^{(1)}(x)=a_j(\lambda, x, y_1, \cdots, y_n)$, $j=1, \cdots, n$, integer $p\geqq 0$ (λ a complex parameter; x on a real interval Δ), have been studied under the hypothesis that the second members are analytic in $\lambda, y_1, \cdots, y_n$ at $(\lambda=\infty, y_1=0, \cdots, y_n=0)$ for x in Δ, differentiable in x. The investigation for (C) has been given for the vicinity of the singular point $\lambda=\infty$. The author's papers on linear differential equations in the Acta Mathematica (vol. 62, pp. 167–226; vol. 67, pp. 1–50) were of much utility in this investigation. A specific method recently developed by the author for non-linear problems (cf. a paper in Compositio Mathematica, vol. 5, pp. 1–66, and

a forthcoming work in the Mémorial des Sciences Mathématiques) has been widely used in this work. The present paper will appear in the Transactions of this Society. (Received July 31, 1937.)

365. Professor A. W. Tucker: *Degenerate cycles bound.*

In topological discussions involving singular chains it has been customary to suppress the degenerate elements which necessarily arise from the use of transformations of a simplicial nature. The following elementary argument shows that this suppression is unnecessary for purposes of homology. Any degenerate cycle on a simplicial complex can be expressed as a sum of cycles eC, where e is a degenerate simplex each vertex of which is repeated an even number of times and C is a wholly non-degenerate cycle. But $aeC \to eC$ where a is a vertex of e. Hence the degenerate cycles fall in the group of bounding cycles. (Received August 4, 1937.)

366. Dean G. D. Birkhoff: *An extensive class of analytic functions.*

This paper will appear shortly in the Annales de l'Institut Henri Poincaré. (Received August 31, 1937.)

367. Professor O. E. Glenn: *The generalized law of Bode as a mathematical principle.*

Bode's law, which expresses the mean distances (r) of our planets from the sun by the formula $ar = z^{\lambda} + b$, was known to Kepler and to Bode in the special, inaccurate form $a = 1/3$, $b = 4/3$, $z = 2$, $\lambda = 0, 1, \cdots, 7$. The author gives the first justification, on theoretical grounds, of the general formula as related to central systems. The mathematical principle reaches the following degree of generality. Let $f(\cdots, d^n r/d\theta^n, \cdots) = 0$, $(n = i, i-1, \cdots, 0)$, be an equation rational and integral in r and in the derivatives, and otherwise numerical and real. Suppose $\theta = q(r_1, r_2) \equiv \alpha r_1^m + \beta r_1^{m-1} r_2 + \cdots + \kappa r_2^m$ represents, parabolically, integral curves in a finite region, and let $\phi(r_1, r_2)$ be a covariant of q under the general linear, real transformation T. One can transform $f = 0$ rationally so that $\phi(r, 1)$ becomes the dependent variable. If s is an integral curve of the new equation and σ the radial coordinate of a point of s, the equation $\phi(r, 1) = \sigma$ is invariant under cases T_j of T. This fact gives information concerning the roots, that is, the intersections, with s, of the integral curves of $f = 0$. In the example where ϕ is the central force for stable orbits, T being special, the roots of $\phi = \sigma$ are a sequence $t, zt, z^2 t, \cdots, z^k t, \cdots$, which may be identified with the sequence of Bode. (Received August 21, 1937.)

368. Professor E. L. Dodd: *Some properties of the general or substitutive mean.*

The function $F(t_1, t_2, \cdots, t_k, u_1, u_2, \cdots, u_n)$ will be called a mean of the u's if for each set of the t's and c for which F is defined, $F(t_1, t_2, \cdots, t_k, c, c, \cdots, c) = c$, it being assumed that there is at least one such set. This mean is transitive; but it need not be internal, translative, homogeneous, symmetrical, or associative. Let $f(x_1, \cdots, x_n, y_1/x_1, \cdots, y_n/x_n)$ be defined for all sets of x's none of which is zero, and let f be a mean of the ratios y_i/x_i. Let $\overline{X} = \phi(X_1, \cdots, X_n)$ be a single-valued homogeneous mean of $X_1, \cdots, X_n$,

such that for at least one set of X_i's, $X_i - \overline{X} \neq 0$. (Homogeneity requires that $\phi(cX_1, \cdots, cX_n) = c\phi(X_1, \cdots, X_n)$. And let $\overline{Y} = \phi(Y_1, \cdots, Y_n)$. When $X_i \neq 0$ and $X_i - \overline{X} \neq 0$ define the function g as follows: $g(X_1, \cdots, X_n, Y_1/X_1, \cdots, Y_n/X_n) = f\{X_1 - \overline{X}, \cdots, X_n - \overline{X}, (Y_1 - \overline{Y})/(X_1 - \overline{X}), \cdots, (Y_n - \overline{Y})/(X_n - \overline{X})\}$. Then g is a mean of the ratios Y_i/X_i. (Received August 31, 1937.)

369. Dr. Aaron Fialkow: *Hypersurfaces of a space of constant curvature.*

The following characterization of spaces of constant curvature by means of their hypersurfaces is proved: When none of the lines of curvature at a point of a hypersurface V_n of a V_{n+1} of constant curvature is tangent to a null vector, their tangents are Ricci principal directions for the V_n. Conversely, if every hypersurface of a $V_{n+1}(n \geqq 3)$ has this property, the V_{n+1} is a space of constant curvature. For $n=2$, every surface V_2 in any Riemann space V_3 has the above property. The paper also gives a complete classification of the Einstein spaces V_n which may be imbedded in a V_{n+1} of constant curvature so that none of the lines of curvature of V_n is tangent to a null vector. This generalizes a recent theorem of T. Y. Thomas and E. Cartan. (Received August 27, 1937.)

370. Dr. Caroline A. Lester: *A determination of the automorphisms of certain algebraic fields.*

In this paper all automorphisms are explicitly obtained as rational functions of one root of the defining equation for the cyclic cubic, quartic, and sextic, the quartic with the four-group, the normal sextic with the symmetric group, and the octics with the abelian groups of types (2, 2, 2) and (4, 2). The determination of the parametric representations of the most general equations defining these fields forms an integral part of the determination of the automorphisms. The computation in the cases $n=6$ and $n=8$ is brought within practicable limits by a free use of matric theory, in particular of a theorem of Williamson (this Bulletin, vol. 37 (1931), p. 585). This has been used in the concluding section to obtain from the results of Albert (Transactions of this Society, vol. 35 (1935), p. 949) a one-parameter family of cyclic octics over the rational field together with their automorphisms. (Received August 30, 1937.)

371. Mr. R. E. O'Connor: *The quaternion congruence* $\overline{\tau}\alpha\tau \equiv \beta \pmod{p}$.

Let $\alpha = [a_1, a_2, a_3]$ and $\beta = [b_1, b_2, b_3]$ be pure, integral quaternions; p an odd prime; p dividing $N\alpha$ and $N\beta$ but not α, β. Gordon Pall has shown (see abstract 43-7-303) that the congruence $\overline{\tau}\alpha\tau \equiv \beta(p)$ is, for given α, solvable in integral quaternions τ for precisely $(p^2-1)/2$ (non-zero) incongruent quaternions β. A criterion is now given for these residues β. By expanding $a_i(\overline{\tau}\alpha\tau)_i$ and setting $g = a_1b_1 + a_2b_2 + a_3b_3$, it is shown that β must satisfy either C_1, $p \mid g$, $(2g \mid p) = 1$; or C_2: $p \mid g$, $\beta \equiv m\alpha(p)$, $(m \mid p) = 1$, m being a rational integer. The condition is shown to be sufficient by counting simultaneous (non-zero) solutions β of the congruences $N\beta \equiv 0(p)$, $g \equiv r(p)$, r any rational integer, which yield p or $p-1$ solutions according as r is or is not $\equiv 0(p)$. It follows easily that, if $p \mid g$, $\beta \equiv s\alpha(p)$ with s a rational integer, precisely $(p^2-1)/2$ quaternion residues β(pure, non-zero) satisfy C_1 or C_2. (Received August 30, 1937.)

NEW PUBLICATIONS

PART I. PURE MATHEMATICS

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ATKIN (E. I.). See MILLS (C. N.).

BELZ (M. H.). See MICHELL (J. H.).

BIEBERBACH (L.). Einführung in die konforme Abbildung. (Sammlung Göschen, vol. 768.) 3d edition. Berlin and Leipzig, de Gruyter, 1937. 136 pp.

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DEGOSANG (O.). See ROTHE (R.)

DENJOY (A.). Introduction à la théorie des fonctions de variables réelles. Part I: Aperçu historique. Géométrie des ensembles cartésiens. Fonctions. Continuité et convergence. Dérivation. (Actualités Scientifiques et Industrielles, No. 451; exposés sur les ensembles et fonctions, publiés sous la direction d'Arnaud Denjoy, I.) Paris, Hermann, 1937. 56 pp.

——— Introduction à la théorie des fonctions de variables réelles. Part II: Intégration. Séries trigonométrique. Fonctions quasi-analytiques. Fonctionnelles et fonctions d'ensemble. (Actualités Scientifiques et Industrielles, No. 452; exposés sur les ensembles et fonctions, publiés sous la direction d'Arnaud Denjoy, II.) Paris, Hermann, 1937. 58 pp.

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BARBIER (D.). Les parallaxes dynamiques des étoiles doubles. (Actualités Scientifiques et Industrielles, No. 348; exposés d'astronomie stellaire, VI.) Paris, Hermann, 1936. 33 pp.

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OFFICIAL COMMUNICATIONS

Meetings of the Society have been fixed at the following times and places:

NEW YORK CITY, October 30, 1937.

Abstracts must be in the hands of Associate Secretary T. R. Hollcroft, 531 West 116th St., New York, N. Y., not later than October 2. Abstracts received by the Secretaries not later than September 3 will appear in the September issue of the Bulletin. By invitation of the Program Committee, Professor D. J. Struik will deliver an address entitled *The application of tensor analysis to problems of electrical engineering.*

IOWA CITY, IOWA, November 26–27, 1937.

Abstracts must be in the hands of Associate Secretary M. H. Ingraham, University of Wisconsin, Madison, Wisconsin, not later than October 30; they will appear in the November issue of the Bulletin. By invitation of the program committee, addresses will be delivered by Professor W. J. Trjitzinsky on *Singular point problems in the theory of linear differential equations*, and by Professor A. H. Copeland on *Fundamental concepts of the theory of probability*

PASADENA, CALIFORNIA, November 27, 1937.

Abstracts must be in the hands of Associate Secretary T. M. Putnam, University of California, Berkeley, California, not later than October 30; they will appear in the November issue of the Bulletin.

INDIANAPOLIS, INDIANA, ANNUAL MEETING, December 28–30, 1937.

Abstracts must be in the hands of Associate Secretary T. R. Hollcroft, 531 West 116th St., New York, N. Y., not later than November 23, 1937. Abstracts received by the Secretaries by November 2 will appear in the November issue of this Bulletin. The Annual Gibbs Lecture will be delivered by Professor Charles A. Kraus before a joint session of this Society and the American Association for the Advancement of Science. Professor G. C. Evans will deliver his retiring address as Chairman of Section A of the latter organization at a joint meeting of Section A with this Society and the Mathematical Association of America. There will also be a joint session of this Society with the Association for Symbolic Logic, and one with the Institute of Mathematical Statistics. By invitation of the Program Committee, Professor Alonzo Church will deliver an address entitled *The constructive second number class.*

R. G. D. RICHARDSON, *Secretary of the Society.*

Articles for insertion in the BULLETIN should be addressed to the Managing Editor, D. R. CURTISS, Northwestern University, Evanston, Ill. Reviews should be sent to W. R. LONGLEY, Yale University, New Haven, Conn. Notes should be sent to H. W. KUHN, Ohio State University, Columbus, Ohio.

Subscriptions to the BULLETIN, orders for back numbers, and inquiries in regard to non-delivery of current numbers should be addressed to the American Mathematical Society, 450–459 Ahnaip St., Menasha, Wis., or Low Memorial Library, 531 West 116th St., New York, N. Y.

The initial fees and the annual dues of members of the Society (see this BULLETIN, p. 322, May, 1930; and the List of Officers and Members, September, 1936, p. 67) are payable to the Treasurer of the Society, Professor P. A. Smith, Low Memorial Library, 531 West 116th St., New York, N. Y.

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